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STATISTICAL CONFIRMATION OF A STELLAR UPPER MASS LIMIT

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ABSTRACT

We derive the expectation value for the maximum stellar mass ($m_{\rm max}$) in an ensemble of N stars, as a function of the IMF upper-mass cutoff ($m_{\rm up}$) and N. We statistically demonstrate that the upper IMF of the local massive star census observed thus far in the Milky Way and Magellanic Clouds clearly exhibits a universal upper mass cutoff around $120-200\ {\rm M}_\odot$ for a Salpeter IMF, although the result is more ambiguous for a steeper IMF.

Subject headings: stars: early-type — stars: fundamental parameters — stars: mass function — stars: statistics — open clusters and associations — galaxies: stellar content

1. INTRODUCTION

The upper mass limit to the stellar initial mass function (IMF) is a critical parameter in understanding stellar populations, star formation, and massive star feedback in galaxies. To date, the largest empirical mass estimate for an individual star is around $200-250\ {\rm M}_\odot$ for the Pistol Star (Figer et al. 1998) near the Galactic Center, and around $120-200\ {\rm M}_\odot$ for the most massive stars in the Large Magellanic Cloud (e.g., Massey & Hunter 1998). In practice, most applications assume an upper mass limit to the IMF of $m_{\rm up}\sim 100$ to $150\ {\rm M}_\odot$. However, there is some confusion on whether the apparent observed upper limit simply represents a statistical limit owing to a lack of sampled stars in individual clusters (Massey 2003; Massey & Hunter 1998; Elmegreen 1997).

Before the advent of the *Hubble Space Telescope* (*HST*), stars with extremely high masses $\gtrsim 1000\ {\rm M}_\odot$ were suggested to exist. The dense stellar knot R136a in the 30 Doradus star-forming region of the LMC was the best-known candidate for harboring such a star (Cassinelli, Mathis, & Savage 1981). The viable candidates for these supermassive stars were eventually resolved by *HST* and ground-based imaging into smaller stars within the conventionally observed mass range (e.g., Weigelt et al. 1991; Heydari-Malayeri, Remy, & Magain 1988). In recent years, however, the possibility of supermassive stars is receiving renewed attention as a possible mode of star formation in the early universe (e.g., Bond, Arnett, & Carr 1984; Larson 1998; Bromm, Kudritzki & Loeb 2001).

It is therefore important to clarify expectations for the highest-mass stars compared to the existing observations. Elmegreen (2000) quantitatively demonstrates that, in the absence of an upper-mass cutoff, stellar masses should be observed up to $40,000\ {\rm M}_\odot$ for the entire Milky Way, based on estimates for the current star formation rate and molecular gas mass. Here, we derive the behavior of the expectation values for the most massive stars and demonstrate that, for a universal IMF, current observations indeed show the existence of an upper-mass limit around $m_{\rm up}\sim 120-200\ {\rm M}_\odot$.

2. THE EXPECTATION VALUE $\langle m_{\rm max}\rangle$

Because of the decreasing power law form of the IMF, the characteristic mass of the largest star formed in clusters of N stars decreases as N decreases. Figure 1 demonstrates this effect with a Monte Carlo simulation. N is drawn for individual star clusters from the universal power-law distribution in N (e.g., Oey & Clarke 1998; Elmegreen & Efremov 1997):

$$n(N)\ dN \propto N^{-2}\ dN \quad , \tag{1}$$

and the stellar masses for each cluster of N stars is drawn from the Salpeter (1955) IMF, within a mass range of 20 to $100\ {\rm M}_\odot$:

$$\phi(m)\ dm \propto m^{-2.35}\ dm \quad . \tag{2}$$

Figure 1 shows the distribution of the most massive star in each cluster, $m_{\rm max}$ vs $\log N$. For single stars, we confirm that the bin of $\log N = 0$ is described simply by the IMF (equation 2). It is apparent that for large N, one can expect that $m_{\rm max}\simeq m_{\rm up}$, but that for small N, the typical most massive star is much lower in mass. For $N=1$, the typical $m_{\rm max}$ is the mean of the IMF, which is $37\ {\rm M}_\odot$ for the distribution of $20\le m\le 100\ {\rm M}_\odot$ used in Figure 1.

We can analytically derive the expectation value $\langle m_{\rm max}\rangle$ for the most massive star in an ensemble of N stars as follows. For N stars, the probability that all are in the mass range 0 to M is,

$$P(0,M) = \left[\int_0^M \phi(m)\ dm\right]^N \quad , \tag{3}$$

where $\phi(m)$ corresponds to the IMF, i.e., a probability distribution function whose integral is unity. It follows that the probability that all the stars are in the mass range 0 to $M+dM$ is,

$$P(0,M+dM) \simeq \left[\int_0^M \phi(m)\ dm\right]^N + \frac{d}{dM}\left[\int_0^M \phi(m)\ dm\right]^N dM \tag{4}$$

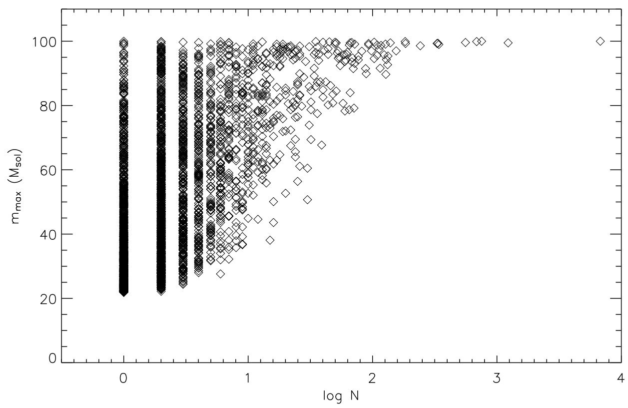

FIG. 1.— Monte Carlo simulation showing the maximum stellar mass $m_{\rm max}$ per cluster vs the number of stars $\log N$ per cluster for 5000 clusters in a distribution of N given by equation 1. A Salpeter IMF is adopted with stellar masses between 20 – 100 $\rm M_\odot$ in this simulation.

by Taylor expansion. Thus we see that the probability that the most massive star is in the range M to $M + dM$ is,

$$P(M, M+dM) = \frac{d}{dM}\left[\int_0^M \phi(m)\ dm\right]^N\ dM \quad , \qquad (5)$$

and the expectation value for the most massive star is,

$$\langle m_{\rm max}\rangle = \int_0^{m_{\rm up}} M\ \frac{d}{dM}\left[\int_0^M \phi(m)\ dm\right]^N\ dM \quad . \qquad (6)$$

Integrating by parts, this yields,

$$\langle m_{\rm max}\rangle = m_{\rm up} - \int_0^{m_{\rm up}} \left[\int_0^M \phi(m)\ dm\right]^N\ dM \quad . \qquad (7)$$

For large N, equation 7 confirms that $\langle m_{\rm max}\rangle \rightarrow m_{\rm up}$, corresponding to an IMF that is well-sampled up to the upper mass limit (termed “saturated” by Oey & Clarke 1998).

We numerically integrate equation 7 using a lower mass limit $m_{\rm lo} = 10\ \rm M_\odot$ instead of 0, and assuming the Salpeter IMF. Figure 2 shows the expectation value for the most massive star $\langle m_{\rm max}\rangle$ vs the upper mass limit $m_{\rm up}$ for N = 100, 250, and 1000 stars (solid lines). The dotted line shows the identical relation $\langle m_{\rm max}\rangle = m_{\rm up}$ for comparison. For lower N, $\langle m_{\rm max}\rangle$ is smaller at any given $m_{\rm up}$, as expected.

3. RESULTS

3.1. *R136a*

We start by comparing Figure 2 to the R136a region in 30 Doradus, which, at an age of 1–2 Myr (Massey & Hunter 1998), is sufficiently young that none of its stars have expired yet as supernovae. We consider stars having $m > 10\ \rm M_\odot$, of which Hunter et al. (1997) found $N = 650$ in this region. We however note that this value represents a strong lower limit, since the star counts are significantly incomplete between 10 and 15 $M_\odot$ (Massey & Hunter 1998). Figure 2 demonstrates that the expectation value of $m_{\rm max}$ is considerably greater than the observed maximum of $\sim 120 - 200\ \rm M_\odot$ in R136a, unless $m_{\rm up}$ is low ($\ll 500\ \rm M_\odot$). Weidner & Kroupa (2004) reached the same conclusion from a similar analysis of R136a; Selman et al. (1999) also suggested a cutoff using a less rigorous analysis. We can furthermore assess the statistical significance of this result by calculating $p(m_{\rm max}|m_{\rm up})$, the probability of obtaining an observed maximum stellar mass $\leq m_{\rm max}$ for a given $m_{\rm up}$. Table 1 lists $p(m_{\rm max}|m_{\rm up})$ calculated from equation 5 for R136a for a range of $m_{\rm up}$. This demonstrates the negligible likelihood ($< 10^{-5}$) that R136a is drawn from a population which extends to 1000 $\rm M_\odot$.

The results of Weidner and Kroupa (2004), and those presented here, do not support the suggestion by Massey (2003) and Massey & Hunter (1998) that the upper IMF in the R136a is consistent with $m_{\rm up} = \infty$. Massey & Hunter (1998) found that the penultimate mass bin in the empirical mass function is fully consistent with the Salpeter slope. However, they omit from their mass function, and from their analysis, stars with inferred masses $> 120\ \rm M_\odot$, because the lack of stellar models in the grid preclude reliable mass determinations. For the two effective temperature scales they adopted, there are 2 or 9 of these omitted, most-massive stars. Although we do not know the exact masses, their numbers are sufficient to determine whether an upper mass cutoff to the IMF power law exists. For a Salpeter IMF, in the absence of an upper mass cutoff, there should be a total of 1.7 times more stars at $m > 120\ \rm M_\odot$ than are found in the the mass bin 85 – 120 $\rm M_\odot$. Massey & Hunter (1998) count (8, 11) stars in the latter mass bin, therefore implying that (14, 19) stars should be found at higher masses. This is significantly more than the (2, 9) stars found. Thus, R136a exhibits a cutoff around $120 - 200\ \rm M_\odot$, consistent with the finding by Weidner & Kroupa (2004).

3.2. *A sample of young OB associations*

Although a truncated IMF in R136a seems conclusively demonstrated, it is possible that the dense, rich cluster environment of this region represents a special case. Can we draw a similar conclusion from a wider sample of ordinary OB associations? To examine this further, we consider the

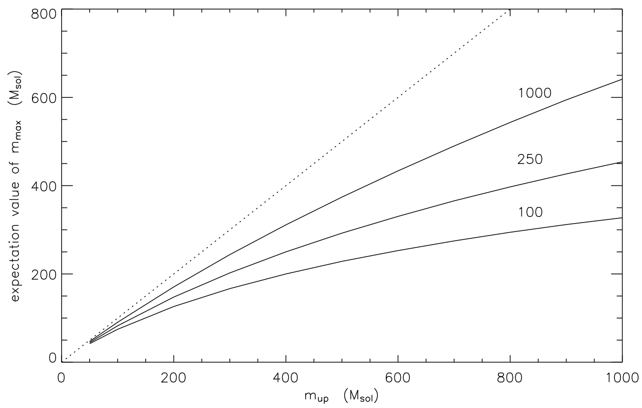

FIG. 2.— The expectation value $\langle m_{\rm max}\rangle$ vs upper mass limit $m_{\rm up}$, for $N = 100,\ 250$, and 1000 stars having masses above $m_{\rm lo} = 10\ {\rm M}_\odot$, assuming a Salpeter IMF. The dotted line shows $m_{\rm max} = m_{\rm up}$ for comparison.

upper IMF from the substantial sample of OB associations that have been uniformly studied by Massey and collaborators, who estimated stellar masses from spectroscopic classifications. Massey, Johnson, & DeGioia-Eastwood (1995) tabulate the numbers of stars having $m \geq 10\ {\rm M}_\odot$ in the Milky Way and LMC associations. To minimize the possibility that the most massive stars have already expired as supernovae, we count only stars in OB associations with ages ≤ 3 Myr. Table 1 shows the observed $N(\geq 10\,{\rm M}_\odot)$ and $m_{\rm max}$ for these objects.

We now compute $p(m_{\rm max}|m_{\rm up})$ for all the objects (Table 1). These show that, although none of these regions individually provide strong constraints on the upper mass cutoff, they collectively point to a conclusion similar to that found for R136a. The total $N = 263$ stars, for which inspection of Figure 2 again shows that the observed maximum stellar masses imply that $m_{\rm up}$ should not exceed a few hundred ${\rm M}_\odot$. Elmegreen (2000) reached a similar conclusion based on the lack of supermassive stars in the entire population of the Milky Way. In considering the total of 263 stars, or Milky Way population, we assume that the IMF is a universal probability distribution function that is independent of specific conditions in individual clusters and parent molecular clouds. Indeed, the IMF is conventionally treated as a universal function (see, e.g., Elmegreen 2000). We also emphasize that our total counts of N are conservative lower limits, since additional young massive stars can be counted from associations studied by other authors. We chose not to include these additional stars in the interest of maintaining a uniform and well-understood sample.

Furthermore, we can now evaluate the total probabilities P that the values of $p(m_{\rm max}|m_{\rm up})$ represent uniform distributions between 0 and 1, as expected for any universal $m_{\rm up}$. For example, we would expect 10% of the regions to fall into the category where $p(m_{\rm max}|m_{\rm up})$ was ≤ 0.1, 20% to have a $p(m_{\rm max}|m_{\rm up})$ of ≤ 0.2, and so on. Figure 3 shows, for each assumed $m_{\rm up}$ of the parent IMF, the distribution of $p(m_{\rm max}|m_{\rm up})$ for the individual regions. It is evident that for higher values of $m_{\rm up}$, the values of $p(m_{\rm max}|m_{\rm up})$ are unacceptably clustered towards small values. A K-S test confirms this conclusion, yielding probabilities that these values are uniformly distributed, of $P < 0.002$, < 0.02, < 0.12, and < 0.47 for, respectively, $m_{\rm up} = 10^4,\ 200,\ 150$, and $120\ {\rm M}_\odot$. We also compute P for an adopted observed $m_{\rm max} = 200\,{\rm M}_\odot$, as might be possible for Tr 14/16 and R136a (Table 1). Figure 4 shows the respective results in this case: $P < 0.002$, < 0.002, 0.47, and < 0.92 for $m_{\rm up} = 10^4,\ 10^3,\ 200$, and $150\ {\rm M}_\odot$. We therefore see that $m_{\rm up} = \infty$, and even $10^3\ {\rm M}_\odot$, are effectively ruled out. *Hence the results from this wider total sample of OB associations points to an upper-mass limit to the IMF around the observed values of* $120 - 200\ {\rm M}_\odot$.

4. CONCLUSION

We have analyzed the upper IMF in a sample of young, nearby OB associations that best represents stellar census data in this regime. The clusters are young enough that their highest-mass members remain present, and the stellar masses are spectroscopically determined by Massey and collaborators (Massey et al. 1995; Massey & Hunter 1998). Our results provide clear evidence for an upper truncation in the IMF. While this result has been previously noted by Weidner & Kroupa in the case of R136a, we show here that it also applies to a much wider sample of OB associations. We have furthermore quantified the statistical significance of such statements. For example, we find that the probability that the stellar population of R136a is drawn from a parent distribution having $m_{\rm up}= 10^4\ {\rm M}_\odot$ is $< 10^{-5}$, and for other associations the probability is only a few percent.

It should be noted that our results are sensitive to the slope of the IMF for stars more massive than $10\ {\rm M}_\odot$. In this mass range, it is often reported that the IMF power-law exponent is close to the Salpeter value ~ -2.35 (e.g., Massey 2003; Schaerer 2003; Kroupa 2002). However, should the slope through some systematic observational bias be significantly steeper, then our demonstration of an upper-mass cutoff becomes less vivid. For example, we find that adopting an IMF slope of -2.8 yields an aggregate probability that the clusters originate from an IMF having $m_{\rm up}= 10^4\ {\rm M}_\odot$ of $P < 0.30$, contrasted to $P < 0.002$ for the Salpeter slope. Conversely, for a parent IMF slope flatter than the Salpeter value, the existence of an upper-mass cutoff is even more strongly demonstrated.

TABLE 1
SAMPLE OF OB ASSOCIATIONS

Name	$N(> 10\ \mathrm{M}_\odot)$	m_{max}	$p(10^4)$	$p(10^3)$	$p(200)$	$p(150)$	$p(120)$
R136a [a]	650	120	10^{-10}	10^{-10}	10^{-5}	0.002	1.000
R136a [b]	650	200	10^{-5}	10^{-5}	1.000	...	...
Berkeley 86	10	40	0.188	0.192	0.224	0.244	0.268
NGC 7380	11	65	0.400	0.409	0.486	0.534	0.592
IC 1805	24	100	0.335	0.350	0.510	0.626	0.784
NGC 1893	19	65	0.206	0.213	0.288	0.338	0.404
NGC 2244	12	70	0.407	0.416	0.502	0.556	0.623
Tr 14/16 [a]	82	120	0.055	0.064	0.231	0.464	1.000
Tr 14/16 [b]	82	200	0.236	0.276	1.000	...	...
LH 10	65	90	0.032	0.037	0.102	0.176	0.324
LH 117/118	40	100	0.161	0.174	0.326	0.458	0.666

[a]Values obtained by adopting 120 $\mathrm{M}_\odot$ for the most massive observed stars.

[a]Values obtained by adopting 200 $\mathrm{M}_\odot$ for the most massive observed stars.

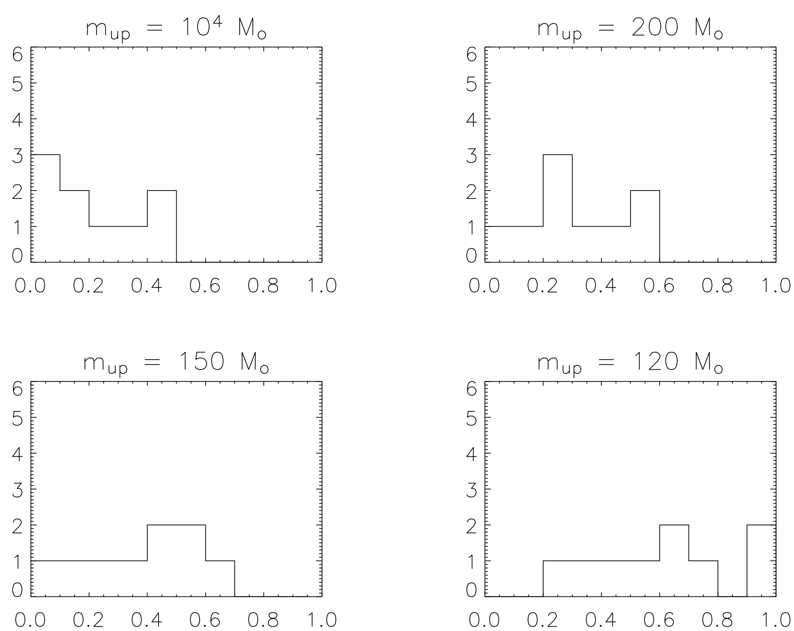

FIG. 3.— Distribution of $p(m_{\mathrm{max}}|m_{\mathrm{up}})$ for the sample of OB associations in Table 1, adopting $m_{\mathrm{max}} = 120\ \mathrm{M}_\odot$ for R136a and Tr 14/16. The aggregate probability that these distributions originate from a uniform distribution are $P < 0.002$, < 0.02, < 0.12, and < 0.47 for, respectively, $m_{\mathrm{up}} = 10^4$, 200, 150, and 120 $\mathrm{M}_\odot$.

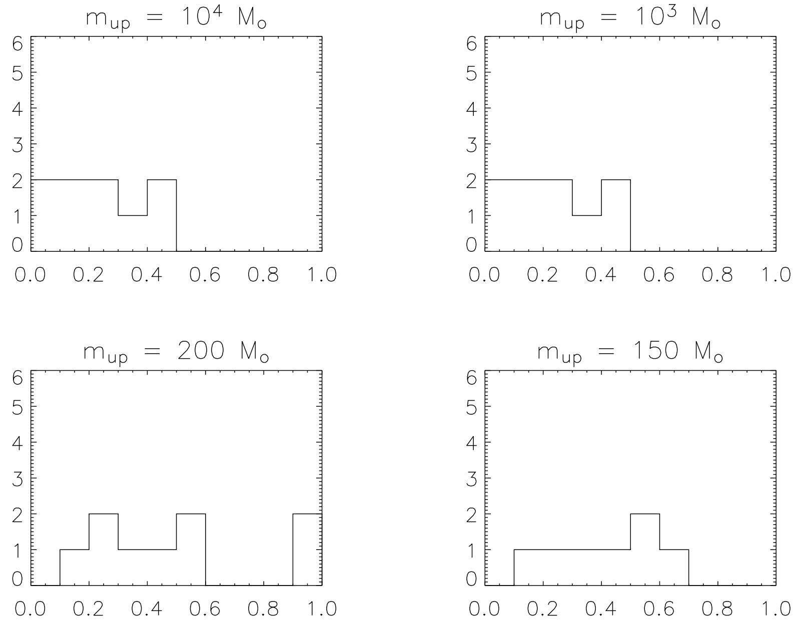

FIG. 4.— Same as Figure 3, but adopting an observed $m_{\rm max}$ for R136a and Tr 14/16 of 200 $M_\odot$. The aggregate probability that these distributions originate from a uniform distribution between 0 and 1 are $P < 0.002$, < 0.002, < 0.47, and < 0.92 for, respectively, $m_{\rm up} = 10^4$, 10^3, 200, and 150 $M_\odot$.

For R136a, $p(10^4) = 6 \times 10^{-4}$ for the steeper slope (*cf.* Table 1), which is still a negligible probability. Weidner & Kroupa (2004) examine the influence of the slope in more detail.

Thus, given the standard Salpeter slope for massive stars, it is hard to escape the conclusion that the IMF is truncated near $m_{\rm up} \sim 120 - 200$ $M_\odot$, based on this analysis. If these results are real, the only other possibilities are that the IMF is not universal, or there is an extreme selection effect that prevents our observations of the most massive stars. We note that $m_{\rm up}$ need not be an absolute limit, but represents at least a dramatic drop from the power-law form of the IMF. Our conclusion depends on the assumption that the highest mass stars have not already expired, and it therefore depends critically on the reliability of evolutionary models for the most massive stars, and on the reliability with which one can assign ages to OB associations. It also assumes that the stars in the OB associations are coeval. Should the star formation process indeed be suppressed at high masses, as suggested by our results, a major goal for theorists will be to identify the physics, e.g., plausibly associated with stellar feedback, that introduces this mass scale into the star formation process.

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